

Heuristics

A workshop for Parents by
Greenwood Primary



Workshop Objective

To create awareness among parents of the different types of heuristics taught in schools



Workshop Outline

8.30 am to 8.40 am - Introduction

8.40 am to 8.55 am - Model Drawing

8.55 am to 9.20 am - Hands-on Session 1

9.20 am to 9.40 am - Guess and Check/

Make a Supposition

9.40 am to 10.05 am - Hands-on Session 2

10.05 am to 10.15 am - Systematic Listing

10.15 am to 10.30 am - Hands-on Session 3



Heuristics

Q : What is a heuristic?

A strategy used to solve word problems
e.g. model drawing, guess and check

Q : Why is it important to learn heuristics ?

Mathematics curriculum is centred on
Problem-Solving.



Heuristics covered in workshop

- Model Drawing
- Guess and Check /
Make a Supposition
- Systematic Listing (Ratio/Fraction)

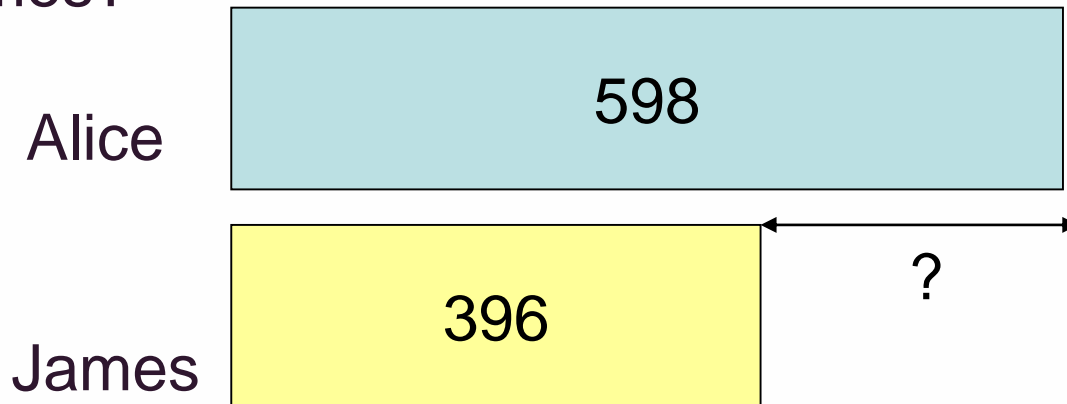


Model Drawing

➤ What is it ?

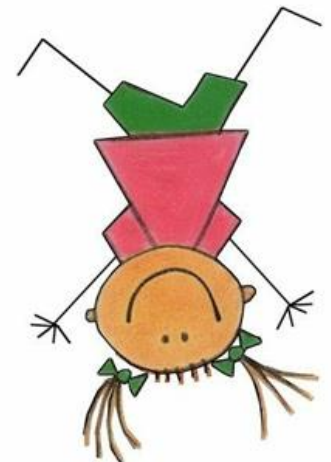
Visual representation of information

Alice has 598 marbles. James had 396 marbles.
How many more marbles does Alice have than James?



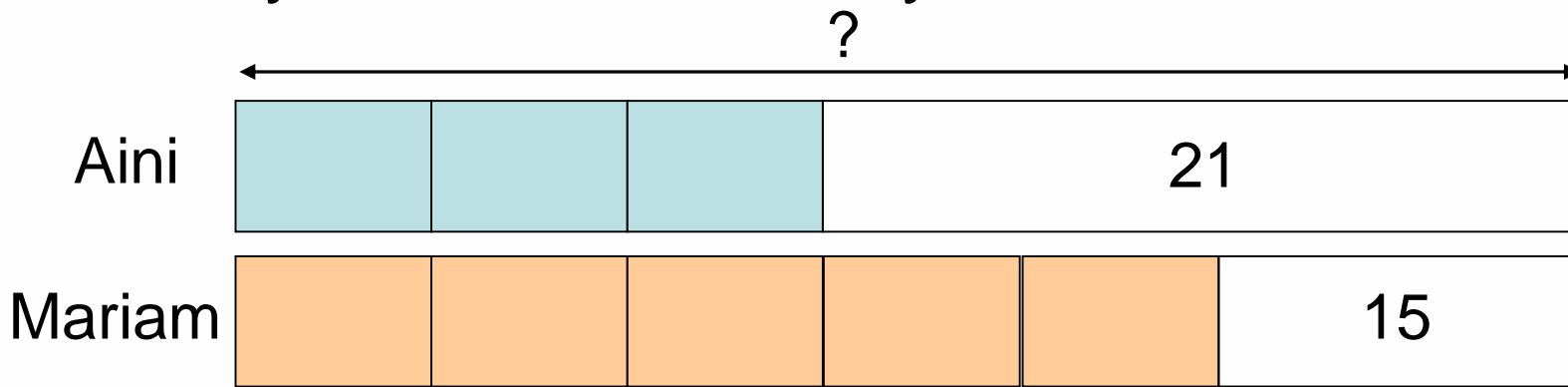
➤ When to use?

Useful tool in solving word problems involving whole numbers, fractions, decimals, ratio and percentage .



Model Drawing (T1)

The ratio of Aini's money to Mariam's money was 3 : 5. After Aini received \$21 and Mariam received \$15 from their aunt, they had the same amount of money. How much money did Aini have in the end?



$$2 \text{ units} = 21 - 15$$

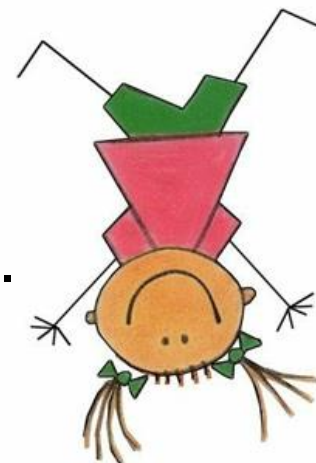
$$21 + 9 = 30$$

$$2 \text{ units} = 6$$

$$1 \text{ unit} = 3$$

$$3 \text{ units} = 9$$

Aini had \$30 in the end.

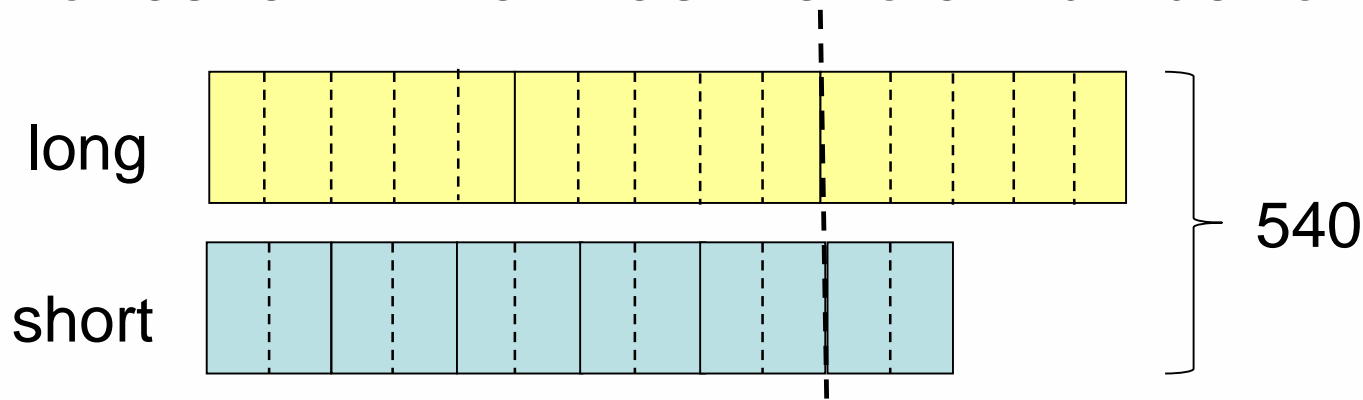


Model Drawing (T2)

Mr Lim had a total of 540 long and short rulers.
After selling an equal number of both types, he

had $\frac{1}{3}$ of the long rulers and $\frac{1}{6}$ of the short

ones left. What was the total number of rulers left?

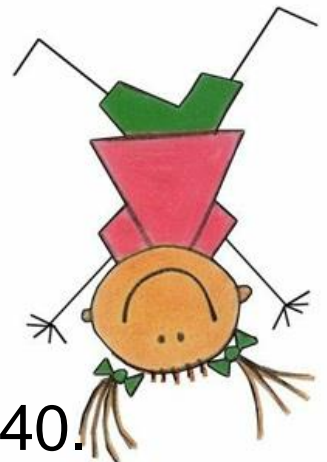


$$15 \text{ units} + 12 \text{ units} = 27 \text{ units}$$

$$27 \text{ units} = 540$$

$$1 \text{ unit} = 20$$

$$7 \text{ units} = 140 \text{ The total number of rulers left was } 140.$$

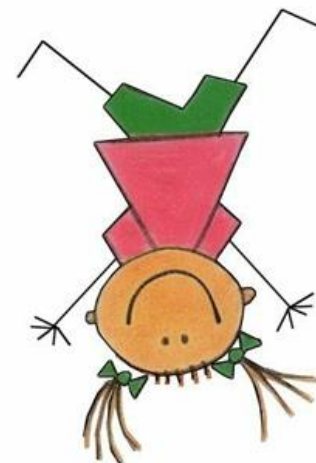
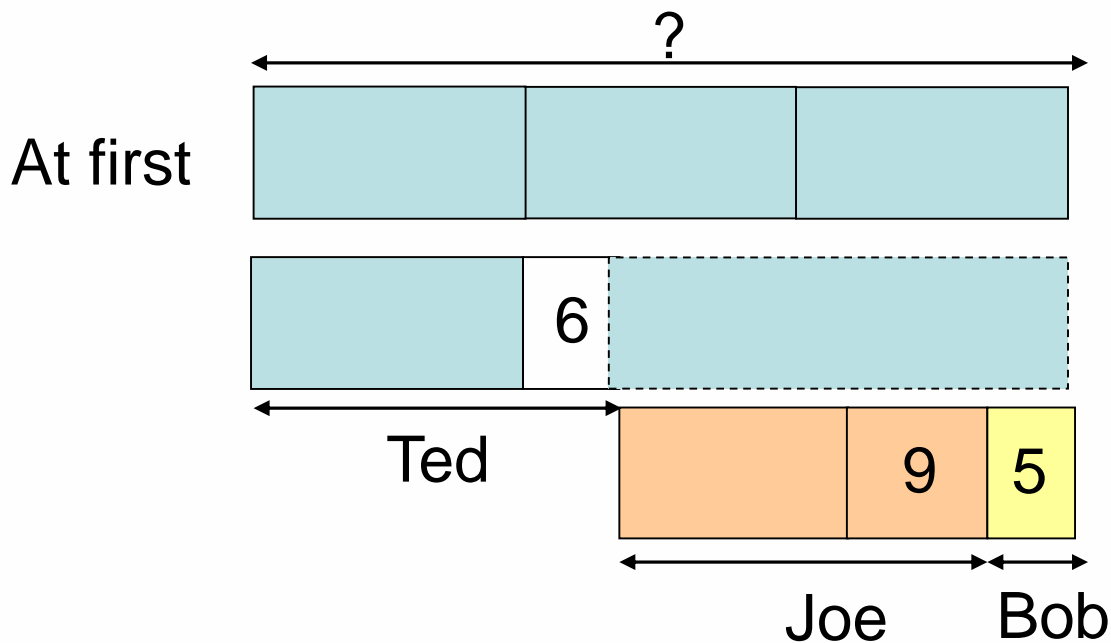


Model Drawing (T3)

1

Ted, Joe and Bob shared a box of apples. Ted took $\frac{1}{3}$ of the apples and another 6 apples from the box.

Joe took $\frac{1}{2}$ of the remaining apples and another 9 apples from the box. Bob took the last 5 apples. How many apples were there in the box at first?



Model Drawing (T3)

Ted, Joe and Bob shared a box of apples. Ted took $\frac{1}{3}$ of the apples and another 6 apples from the box. Joe took $\frac{1}{2}$ of the remaining apples and another 9 apples from the box. Bob took the last 5 apples. How many apples were there in the box at first?

$$9 + 5 = 14$$

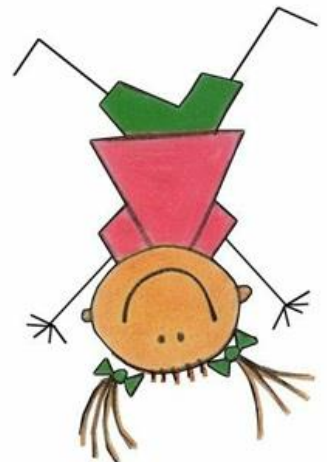
$$14 \times 2 = 28$$

$$28 + 6 = 34$$

$$34 \div 2 = 17$$

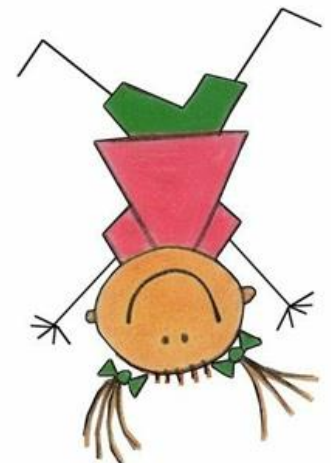
$$17 \times 3 = 51$$

There were 51 apples in the box at first.



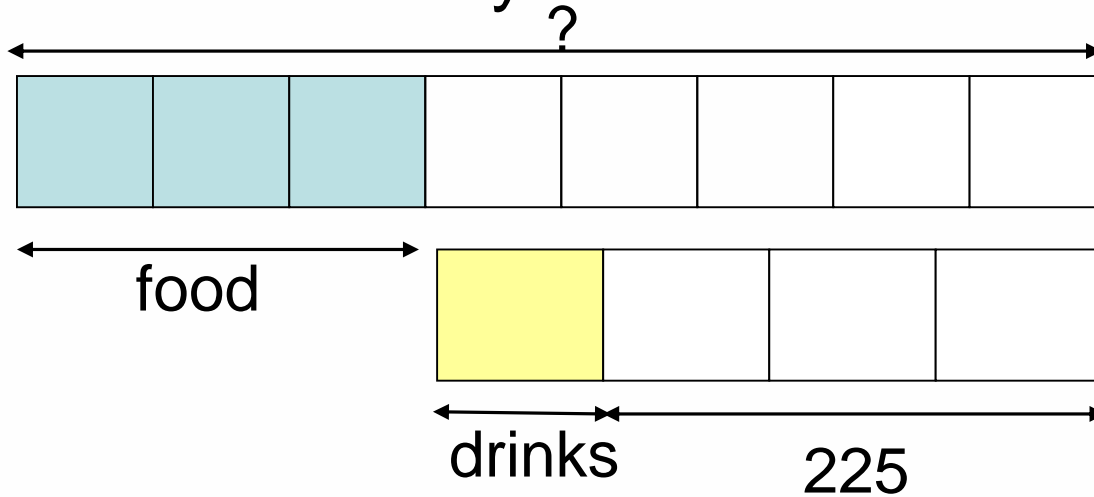


Hands-on Session 1 (15 min)



Model Drawing (H1)

Jack spent $\frac{3}{8}$ of his money on food and $\frac{1}{4}$ of his remaining money on drinks. If he had \$225 left, how much money did he have at first?

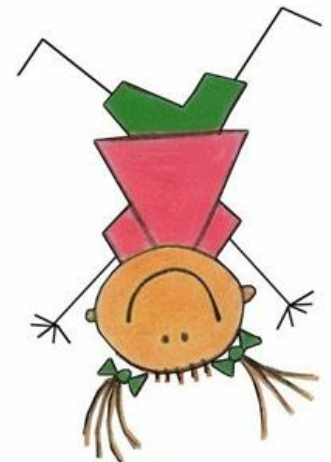


$$3 \text{ units} = 225 \quad 5 \text{ parts} = 300$$

$$1 \text{ unit} = 75 \quad 1 \text{ part} = 60$$

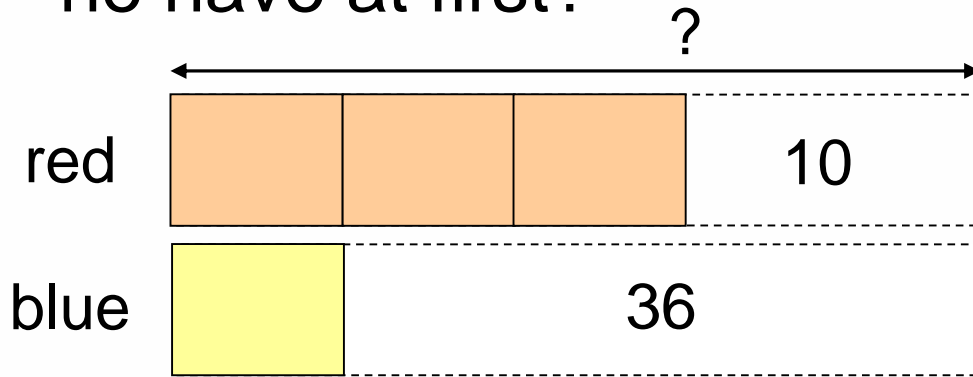
$$4 \text{ units} = 300 \quad 8 \text{ parts} = 480$$

He had \$480 at first.



Model Drawing (H2)

John had the same number of red and blue marbles at first. During a game, he lost 10 red and 36 blue marbles. The ratio of the red and blue marbles became 3:1. How many blue marbles did he have at first?

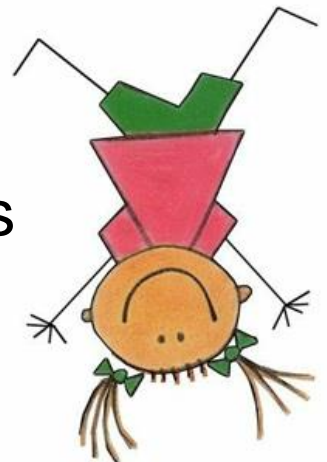


$$\begin{aligned} 2 \text{ units} &= 36 - 10 \\ &= 26 \end{aligned}$$

$$\begin{aligned} 1 \text{ unit} &= 26 \div 2 \\ &= 13 \end{aligned}$$

$$13 + 36 = 49$$

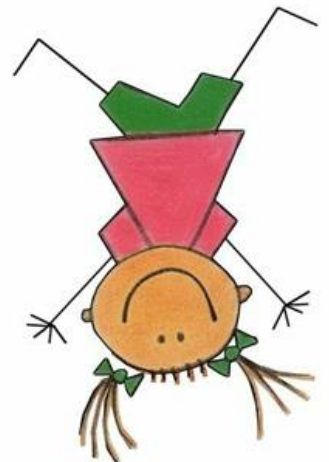
He had 49 blue marbles at first.



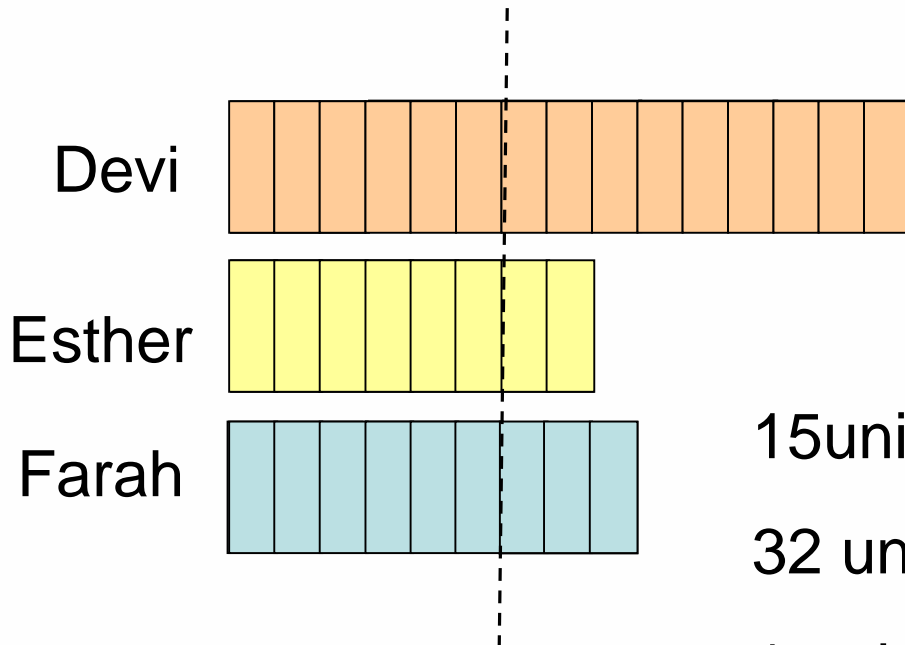
Model Drawing (H3)

Three girls used the same number of beads to make necklaces. Devi used $\frac{2}{5}$ of her beads, Esther used $\frac{3}{4}$ of hers and Farah used $\frac{2}{3}$ of hers. They had a total of 1440 beads at first.

How many beads did each girl use? **Past PSLE Q**



Model Drawing (H3)



$$15 \text{ units} + 8 \text{ units} + 9 \text{ units} = 1440$$

$$32 \text{ units} = 1440$$

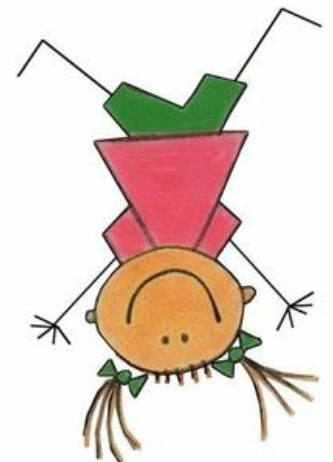
$$1 \text{ unit} = 1440 \div 32$$

$$= 45$$

$$6 \text{ units} = 6 \times 45$$

$$= 270$$

Each girl used 270 beads.



Guess and Check

➤ What is it ?

There were 10 cars and motorcycles altogether.

They had 28 wheels in all. How many cars were there?

No. of Cars	Cars (4 Wheels)	No. of Motorcycles	Motorcycles (2 wheels)	Total	Check (34?)
5	$\underline{5} \times 4 = 20$	5	$\underline{5} \times 2 = 10$	$20 + 10 = 30$	x
8	$\underline{8} \times 4 = 24$	2	$\underline{2} \times 2 = 4$	$24 + 4 = 28$	Yes!

6 columns, Headings, starting point and check point

➤ When to use? To meet given conditions or criteria in order to obtain answer



Make a Supposition

- *What*

Make an assumption

- *When*

Two quantities with checkpoint and total number given



Guess and Check/Supposition(T1)

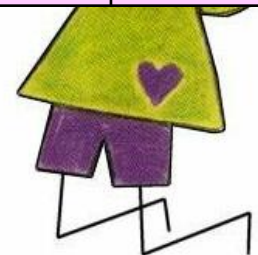
A salesman had to deliver 100 watches to his customers. For every watch he delivers on time, he was paid \$100. He was paid \$80 if he delivered the watch late. If he made \$9940 from the delivery, how many watches were delivered late?



Guess and Check/Supposition(T1)

No. of watches delivered		Cost		Amt Earned	Check
On time	Late	On time	Late		
50	50	$\$100 \times 50$ $= \$5000$	$\$80 \times 50$ $= \$4000$	$\$5000 + \4000 $= \$9000$	x
51	49	$\$100 \times 51$ $= \$5100$	$\$80 \times 49$ $= \$3920$	$\$5100 + \3920 $= \$9020$	x
97	3	$\$100 \times 97$ $= \$9700$	$\$80 \times 3$ $= \$240$	$\$9700 + \240 $= \$9940$	Yes!

3 watches were delivered late.



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Guess and Check/Supposition(T1)

A salesman had to deliver 100 watches to his customers. For every watch he delivers on time, he was paid \$100. He was paid \$80 if he delivered the watch late. If he made \$9940 from the delivery, how many watches were delivered late?

Assume all watches were delivered on time.

$$100 \times \$100 = \$10000$$

$$\$10000 - \$9940 = \$60$$

$$\$100 - \$80 = \$20$$

$$\$60 \div 20 = 3$$

3 watches were delivered late.

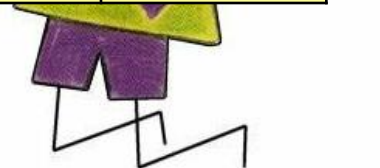


Guess and Check /Supposition (T2)

A school paid \$434 for admission ticket to a concert for 101 pupils and teachers. An adult ticket cost \$6 and a child ticket cost \$4. How many adult tickets did the school buy?

No of pupils	Cost (x \$4)	No. of teachers	Cost (x \$6)	Total	Check
80	\$320	21	\$126	\$446	x
90	\$360	11	\$\$66	\$426	X
86	\$344	15	\$90	\$434	Yes!

The school bought 15 adult tickets.



Guess and Check / Supposition (T2)

A school paid \$434 for admission ticket to a concert for 101 pupils and teachers. An adult ticket cost \$6 and a child ticket cost \$4. How many adult tickets did the school buy?

Assume all tickets are child tickets.

$$\$4 \times 101 = \$404$$

$$\$434 - \$404 = \$30$$

$$\$6 - \$4 = \$2$$

$$\$30 \div \$2 = 15$$

The school bought 15 adult tickets.





Hands-on Session 1 (10 min)



Guess and Check /Supposition (H1)

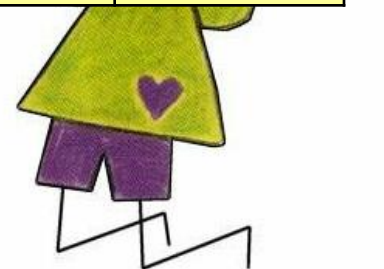
A salesmen had to sell 100 watches for his company. For every watch he sold, he earned \$50.40. He had to pay his company \$110.80 for each unsold watch. If he made \$4234 from the sale, how many watches were unsold?



Guess and Check / Supposition (H1)

No. of watches sold	Amt earned (x \$80)	No. of unsold watches	Amt Paid (x\$110.80)	Total earned (-)	Check
80	\$4032	20	\$2216	\$1816	X
90	\$4536	10	\$1108	\$3428	x
95	\$4788	5	\$554	\$4234	Yes!

5 watches were unsold.



Guess and Check / Supposition (H1)

A salesmen had to sell 100 watches for his company. For every watch he sold, he earned \$50.40. He had to pay his company \$110.80 for each unsold watch. If he made \$4234 from the sale, how many watches were unsold?

Assume all watches are sold.

$$100 \times \$50.40 = \$5040$$

$$\$5040 - \$4234 = \$806$$

$$\$110.80 + \$50.40 = \$161.20$$

$$\$806 \div \$161.20 = 5$$

5 watches were unsold.



Guess and Check/Supposition(H2)

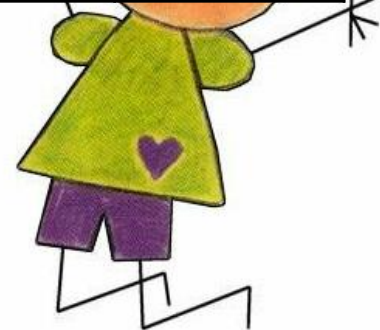
Meng sold a total of 368 large and small durians. He sold the large durians at \$9 each and the small durians at \$5 each. He collected \$2760. How many large durians did Meng sell? **Past PSLE Q**



Guess and Check/Supposition(H2)

No. of large durians	Amt (x \$9)	No. of small durians	Amt (x \$5)	Total	Check
200	\$1800	168	\$840	\$2640	X
210	\$1890	158	\$790	\$2680	x
230	\$2070	138	\$\$690	\$2760	Yes!

Meng sold 230 large durians.



Guess and Check/Supposition(H2)

Meng sold a total of 368 large and small durians. He sold the large durians at \$9 each and the small durians at \$5 each. He collected \$2760. How many large durians did Meng sell?

Assume all are small durians.

$$368 \times \$5 = \$1840$$

$$\$2760 - \$1840 = \$920$$

$$\$9 - \$5 = \$4$$

$$\$920 \div \$4 = 230$$

Meng sold 230 large durians.



Systematic Listing

- *What is it?*

Day	Distance climbed during the day (m)	Distance slid in the night (m)
1	5	3
2	$2 + 5 = 7$	$7 - 3 = 4$
3	$4 + 5 = 9$	$9 - 3 = 6$
4	$6 + 5 = 11$	$11 - 3 = 8$

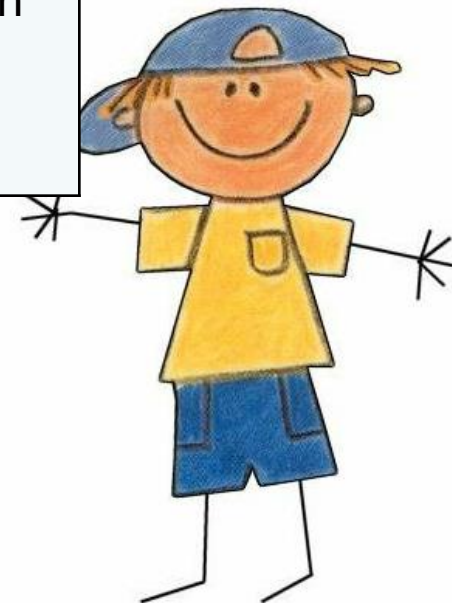
- *When to use?*

There are given conditions or criteria in a problem
e.g. If I give the children 4 marbles each,
I am left with 1 marble.



Systematic Listing Vs Guess and Check

Systematic Listing	Guess and Check
List in order	Logical guess and can skip guesses
Check points such as final ratio	Checkpoint such as total given



Systematic Listing (T1)

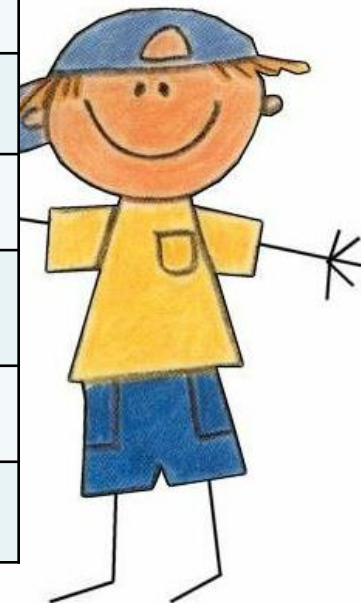
John had the same number of red and blue marbles at first. During a game, he lost 10 red and 36 blue marbles. The ratio of the red and blue marbles became 3:1. How many red marbles did he have at first?



Systematic Listing (T1)

Before		After		
Red	Blue	Red	Blue	Ratio
40	40	$40-10=30$	$40-36=4$	$30:4=15:2$
41	41	$41-10=31$	$41-36=5$	$31:5$
42	42	$42-10=32$	$42-36=6$	$32:6=16:3$
43	43	$43-10=33$	$43-36=7$	$33:7$
44	44	$44-10=34$	$44-36=8$	$34:8=17:4$
45	45	$45-10=35$	$45-36=9$	$35:9$
46	46	$46-10=36$	$46-36=10$	$36:10=18:5$
47	47	$47-10=37$	$47-36=11$	$37:11$
48	48	$48-10=38$	$48-36=12$	$38:12=19:6$
49	49	$49-10=39$	$49-36=13$	$39:13=3:1$

He had 49 red marbles at first.



Systematic Listing (T2)

Eight years ago, the ratio of Mr Lim's age to his brother's age was 4:3. This year, the ratio of Mr Lim's age to his brother's age is 5:4. What will be the ratio of Mr Lim's age to his brother's age in 8 years' time?

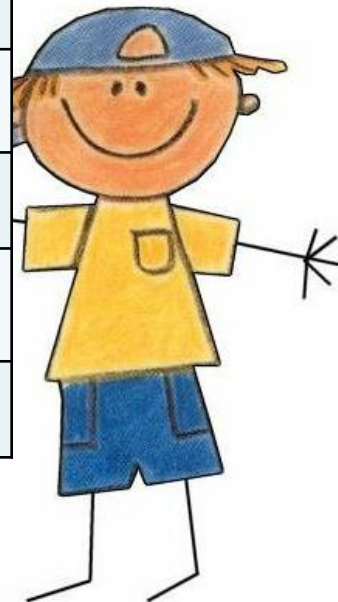
8 years ago		This year		
Mr Lim	Bro	Mr Lim	Bro	Ratio
20	15	$20+8=28$	$15+8=23$	28:23
24	18	$24+8=32$	$18+8=26$	$32:26=16:13$
28	21	$28+8=36$	$21+8=29$	36:29
32	24	$32+8=40$	$24+8=32$	$40:32=5:4$

$$40 + 8 = 48$$

$$32 + 8 = 40$$

$$48 : 40 = 6 : 5$$

The ratio is 6:5.



Systematic Listing (T2)

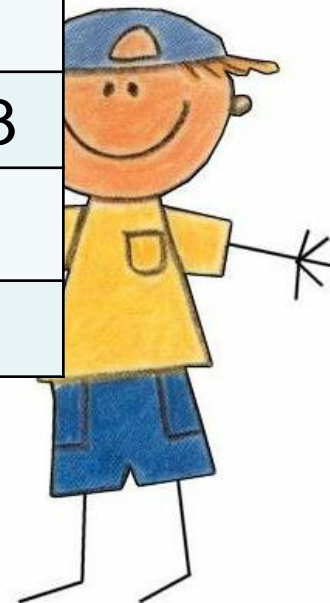
8 years ago		This year		
Mr Lim	Bro	Mr Lim	Bro	Ratio
4	3	$4 + 8 = 12$	$3 + 8 = 11$	12:11
8	6	$8 + 8 = 16$	$6 + 8 = 14$	$16:14 = 8:7$
12	9	$12 + 8 = 20$	$9 + 8 = 17$	20:17
16	12	$16 + 8 = 24$	$12 + 8 = 20$	$24:20 = 6:5$
20	15	$20 + 8 = 28$	$15 + 8 = 23$	28:23
24	18	$24 + 8 = 32$	$18 + 8 = 26$	$32:26 = 16:13$
28	21	$28 + 8 = 36$	$21 + 8 = 29$	36:29
32	24	$32 + 8 = 40$	$24 + 8 = 32$	$40:32 = 5:4$

$$40 + 8 = 48$$

$$32 + 8 = 40$$

$$48:40 = 6:5$$

The ratio is 6:5.





Hands-on Session 1 (10 min)



Systematic Listing (H1)

The ratio of Hafiz's age to his mother's age is 1:3 now. Five years later, Hafiz's age will be $\frac{3}{7}$ that of his mother's age. How much older is Hafiz's mother than Hafiz?



Systematic Listing (H1)

Now		5 years later		
Hafiz	Mother	Hafiz	Mother	Ratio
6	18	$6+5=11$	$18+5=23$	11:23
7	21	$7+5=12$	$21+5=26$	$12:26=6:13$
8	24	$8+5=13$	$24+5=29$	13:29
9	27	$9+5=14$	$27+5=32$	$14:32=7:16$
10	30	$10+5=15$	$30+5=35$	$15:35=3:7$

$$35 - 15 = 20$$

Hafiz's mother is 20 years older than Hafiz.



Systematic Listing (H2)

The ratio of the number of pears to oranges that James had at first was 2:5. After selling 11 pears and 21 oranges, the ratio of the number of pears to oranges became 1:3. How many oranges did James have at first?



Systematic Listing (H2)

Before (2:5)		After		
Pears	Oranges	Pears (-11)	Oranges (-21)	Ratio (1:3)
12	30	1	9	1:9
14	35	3	14	3:14
16	40	5	19	5:19
18	45	7	24	7:24
20	50	9	29	9:29
22	55	11	34	11:34
24	60	13	39	13:39=1:3

James had 60 oranges at first.

Q & A



Tips to become better problem solvers

- ❖ Practice makes perfect !
- ❖ Do not be afraid of making mistakes.
- ❖ Learn together with your child.
- ❖ Encourage your child to persevere.
- ❖ Build confidence in your child.



Thank You !

